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Cosmological singleton gravity theory and dS/LCFT correspondence

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ABSTRACT: We study the evolution of cosmological perturbations generated during de Sitter inflation in the singleton gravity theory. This theory is composed of a dipole pair in addition to tensor. We obtain the singleton power spectra which show that the de Sitter/logarithmic conformal field theory (dS/LCFT) correspondence works for computing the power spectra in the superhorizon limit. Also we compute the spectral indices for light singleton which contains a logarithmic correction.

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1 Introduction

The singleton theory is quite interesting because it provides two coupled scalar equations which are combined to yield the degenerate fourth-order equation which is the same equation for the degenerate Pais-Uhlenbeck oscillator [1]. The Dirac quantization of the Pais-Uhlenbeck oscillator was carried out in [2, 3]. In the anti-de Sitter (AdS) literature, this describes a dipole pair field (singleton) of the AdS group [4]. Later on, this theory was used widely to derive the AdS/logarithmic conformal field theory (LCFT) correspondence [5–8] and the de Sitter (dS)/LCFT correspondence [9]. In other words, the singleton action on the AdS/dS background is a bulk action to derive the LCFT [10, 11] on its boundary. Explicitly, a dipole pair (φ_1, φ_2) on AdS/dS space are dual to the rank-2 LCFT with two operators (σ_1, σ_2) .

On the other hand, the detection of primordial gravitational waves by BICEP2 [12] has indicated that the cosmic inflation occurred at a high scale of 10^{16} GeV. A single scalar field (inflaton) is still known to be a promising model for describing the slow-roll (dS-like) inflation [13, 14]. An important issue to be resolved indicates that the tensor-to-scalar ratio is given by $r = 0.2^{+0.07}_{-0.05}$ (considering the dust reduction, it reduces to $r = 0.16^{+0.06}_{-0.05}$) which is outside of the 95% confidence level of the Planck measurement [15]. Accordingly, many literature have provided plausible ways to reduce the tension between BICEP2 and Planck measurement [16–22]. Also, it is meaningful to mention recent claims that the entire signal may be due to polarized dust emission [23–25].

The dS/CFT correspondence has predicted the form of the three-point correlator of the operator which is dual to the inflaton perturbation generated during slow-roll inflation [26]. This dual correlator was related closely to the three-point correlator of the curvature perturbation generated during slow-roll inflation. Importantly, this correspondence has provided the first derivation of the non-Gaussianity from the single field inflation.

Hence, it is quite interesting to compute the power spectrum of singleton (other than inflaton) generated during dS inflation because its equation is a degenerate fourth-order equation. In order to compute the power spectrum, one needs to choose the Bunch-Davies vacuum in the subhorizon limit of $z \rightarrow \infty$. Therefore, one has to quantize the singleton canonically as we do for the inflaton. Also, it is important to see whether the dS/LCFT correspondence plays a crucial role in computing the power spectrum in the superhorizon limit of $z \rightarrow 0$ [9]. As far as we know, there is no direct evidence for the dS/LCFT correspondence. We will show that the momentum LCFT-correlators $\langle \sigma_a(k) \sigma_b(-k) \rangle$ obtained from the extrapolation approach take the same form as the power spectra $[\mathcal{P}_{ab,0}(k, -1)] \times k^{-3}$. This shows that the dS/LCFT correspondence works well for obtaining the power spectra in the superhorizon limit.

2 Singleton gravity theory

Let us first consider the singleton gravity theory where a dipole pair ϕ_1 and ϕ_2 are coupled minimally to Einstein gravity. The action is given by

$$S_{\text{SG}} = S_{\text{E}} + S_{\text{S}} = \int d^4x \sqrt{-g} \left[\left(\frac{R}{2\kappa} - \Lambda \right) - \left(\partial_\mu \phi_1 \partial^\mu \phi_2 + m^2 \phi_1 \phi_2 + \frac{\mu^2}{2} \phi_1^2 \right) \right], \quad (2.1)$$

where the first two terms are introduced to provide de Sitter background with $\Lambda > 0$ and the last three terms (S_{S}) represent the singleton theory composed of two scalars ϕ_1 and ϕ_2 [5–7]. Here we have $\kappa = 8\pi G = 1/M_{\text{P}}^2$, M_{P} being the reduced Planck mass and m^2 is the degenerate mass-squared for the singleton. We stress that S_{SG} denotes the action for the singleton gravity theory, whereas S_{S} is the action for the singleton theory itself.

The Einstein equation takes the form

$$G_{\mu\nu} + \kappa \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \quad (2.2)$$

with the energy-momentum tensor

$$T_{\mu\nu} = 2\partial_\mu \phi_1 \partial_\nu \phi_2 - g_{\mu\nu} \left(\partial_\mu \phi_1 \partial^\mu \phi_2 + m^2 \phi_1 \phi_2 + \frac{\mu^2}{2} \phi_1^2 \right). \quad (2.3)$$

On the other hand, two scalar field equations are coupled to be

$$(\nabla^2 - m^2)\phi_1 = 0, \quad (\nabla^2 - m^2)\phi_2 = \mu^2 \phi_1 \quad (2.4)$$

which are combined to give a degenerate fourth-order equation

$$(\nabla^2 - m^2)^2 \phi_2 = 0. \quad (2.5)$$

This reveals the nature of the singleton theory as S_{S} takes the following form upon using (2.4) to eliminate the auxiliary field ϕ_1 [27, 28]:

$$S_{\text{S}} = \frac{1}{2\mu^2} \int d^4x \sqrt{-g} (\nabla^2 - m^2) \phi_2 (\nabla^2 - m^2) \phi_2. \quad (2.6)$$

The solution of dS spacetime comes out when one chooses the vanishing scalars

$$\bar{R} = 4\kappa\Lambda, \quad \bar{\phi}_1 = \bar{\phi}_2 = 0. \quad (2.7)$$

Explicitly, curvature quantities are given by

$$\bar{R}_{\mu\nu\rho\sigma} = H^2(\bar{g}_{\mu\rho}\bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma}\bar{g}_{\nu\rho}), \quad \bar{R}_{\mu\nu} = 3H^2\bar{g}_{\mu\nu} \quad (2.8)$$

with a constant Hubble parameter $H^2 = \kappa\Lambda/3$. We choose the dS background explicitly by choosing a conformal time η

$$ds_{\text{dS}}^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu = a(\eta)^2 \left[-d\eta^2 + \delta_{ij}dx^i dx^j \right], \quad (2.9)$$

where the conformal scale factor is

$$a(\eta) = -\frac{1}{H\eta} \rightarrow a(t) = e^{Ht}. \quad (2.10)$$

Here the latter denotes the scale factor with respect to cosmic time t . During the dS stage, a goes from small to a very large value like $a_f/a_i \simeq 10^{30}$ which implies that the conformal time $\eta = -1/aH$ ($z = -k\eta$) runs from $-\infty$ (the infinite past) to 0^- (the infinite future). The two boundaries ($\partial\text{dS}_{\infty/0}$) of dS space are located at $\eta = -\infty$ together with a point $\eta = 0^-$ which make the boundary compact [26]. It is worth noting that the Bunch-Davies vacuum will be chosen at $\eta = -\infty$, while the dual (L)CFT can be thought of as living on a spatial slice at $\eta = 0^-$.

We choose the Newtonian gauge of $B = E = 0$ and $\bar{E}_i = 0$ for cosmological perturbation around the dS background (2.9). In this case, the cosmologically perturbed metric can be simplified to be

$$ds^2 = a(\eta)^2 \left[-(1 + 2\Psi)d\eta^2 + 2\Psi_i d\eta dx^i + \left\{ (1 + 2\Phi)\delta_{ij} + h_{ij} \right\} dx^i dx^j \right] \quad (2.11)$$

with transverse-traceless tensor $\partial_i h^{ij} = h = 0$. Also, one has the scalar perturbations

$$\phi_1 = \bar{\phi}_1 + \varphi_1, \quad \phi_2 = \bar{\phi}_2 + \varphi_2. \quad (2.12)$$

In order to get the cosmological perturbed equations, one linearize the Einstein equation (2.2) directly around the dS

$$\delta R_{\mu\nu}(h) - 3H^2 h_{\mu\nu} = 0 \rightarrow \bar{\nabla}^2 h_{ij} = 0. \quad (2.13)$$

We would like to mention briefly two metric scalars Ψ and Φ , and a vector Ψ_i . The linearized Einstein equation requires $\Psi = -\Phi$ which was used to define the comoving curvature perturbation in the slow-roll inflation and thus, they are not physically propagating modes. In the dS inflation, there is no coupling between $\{\Psi, \Phi\}$ and $\{\varphi_1, \varphi_2\}$ because of $\bar{\phi}_1 = \bar{\phi}_2 = 0$. The vector is also a non-propagating mode in the singleton gravity theory because it has no its kinetic term. The linearized scalar equations are given by

$$\begin{aligned} (\bar{\nabla}^2 - m^2)\varphi_1 &= 0, \\ (\bar{\nabla}^2 - m^2)\varphi_2 &= \mu^2\varphi_1. \end{aligned} \quad (2.14)$$

These are combined to provide a degenerate fourth-order scalar equation

$$(\bar{\nabla}^2 - m^2)^2 \varphi_2 = 0, \quad (2.15)$$

which is our main equation to be solved for cosmological purpose.

3 dS/LCFT correspondence in the superhorizon

First of all, we briefly review what are similarities and differences between AdS/CFT and dS/CFT dictionaries. The first version of the AdS/CFT dictionary was stated in terms of an equivalence between bulk and boundary partition functions in the presence of deformations:

$$Z_{\text{bulk}}[\phi_0, \mathcal{M}] = Z_{\text{CFT}}[\phi_0, \mathcal{O}, \partial\mathcal{M}], \quad (3.1)$$

where on the bulk side ϕ_0 specifies the boundary conditions of bulk field ϕ propagating on \mathcal{M} , whereas on the boundary CFT ϕ_0 denotes the sources of operators \mathcal{O} on the boundary $\partial\mathcal{M}$. Correlator of dual CFT can be computed by differentiating the partition function with respect to the sources and then, setting them to zero as

$$\langle \mathcal{O}(\mathbf{x}) \mathcal{O}(\mathbf{y}) \rangle_{\text{d}} = \frac{\delta^2 Z_{\text{CFT}}}{\delta \phi_0(\mathbf{x}) \delta \phi_0(\mathbf{y})} \Big|_{\phi_0=0}. \quad (3.2)$$

This is called “differentiate” (GKPW) dictionary [29]. The second version consists of computing bulk-to-boundary propagators first and pulling CFT correlators to the boundary as

$$\langle \mathcal{O}(\mathbf{x}) \mathcal{O}(\mathbf{y}) \rangle_{\text{e}} = \lim_{z \rightarrow 0} z^{-2\Delta} \langle \phi(\mathbf{x}, z) \phi(\mathbf{y}, z) \rangle. \quad (3.3)$$

This version was used in [30] and was referred to “extrapolate” (BDHM) dictionary [31].

Concerning correlation functions of a free massive scalar in AdS and dS, the following three statements appear importantly [32]:

- (a) In Euclidean AdS_{d+1} with $\ell_{\text{AdS}}^2 = 1$, either the differentiation of the partition function with respect to sources or extrapolation of the bulk operators to the boundary produce CFT correlators of an operator with dimension $\Delta = \frac{d}{2} + \frac{\sqrt{d^2 + 4m^2}}{2}$.
- (b) In Lorentzian dS_{d+1} with $\ell_{\text{dS}}^2 = 1$, the extrapolated bulk correlators are a sum of two contributions. One is the leading behavior of a CFT correlator of an operator with dimension $d - \delta = \frac{d}{2} - \frac{\sqrt{d^2 - 4m^2}}{2}$, whereas the other comes from the leading behavior of a CFT correlator of an operator with dimension $\delta = \frac{d}{2} + \frac{\sqrt{d^2 - 4m^2}}{2}$.
- (c) In Lorentzian dS_{d+1} with $\ell_{\text{dS}}^2 = 1$, functional derivatives of late-time Schrödinger wavefunction produce CFT correlators with dimension δ only.

The dominant term in (b) was computed by Witten for a particular scalar [33], while a massless version of statement (c) was firstly made by Maldacena [26]. This implies that the dS/CFT “extrapolate” and “differentiate” dictionaries are inequivalent to each other. Particularly, the dimension of CFT operators associated to a massive scalar is different: $\Delta_+(= \delta) = \frac{3}{2} + \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$ for “differentiate” dictionary and both $\Delta_{\pm} = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$ ($\Delta_- = w$) for “extrapolate” dictionary in four dimensional dS space. Accordingly, following (c) to compute cosmological correlator of a massive scalar, it in momentum space is inversely proportional to CFT correlator with dimension Δ_+ as

$$\langle \phi(k) \phi(-k) \rangle \propto \frac{1}{2\text{Re}\langle \mathcal{O}(k) \mathcal{O}(-k) \rangle_{\text{d}}} \propto \frac{1}{k^{-3+2\Delta_+}} = k^{2w-3}, \quad (3.4)$$

which leads to the power spectrum for a massive scalar in the superhorizon limit. If one employs (c) to derive the dS/LCFT correspondence, the approach (c) may break down for

deriving LCFT correlators because all LCFT correlators in AdS_{d+1} were derived based on the extrapolation approach (b) [5–8]. Hence, we wish to use the extrapolation approach (b) to derive the LCFT correlators from the bulk correlators. In this case, the cosmological correlator is directly proportional to the CFT correlator with different dimension Δ_-

$$\langle \phi(k)\phi(-k) \rangle \propto \langle \sigma(k)\sigma(-k) \rangle_e \propto k^{2w-3} \quad (3.5)$$

as was shown in (3.3).

To develop the dS/LCFT correspondence [9], we first solve Eqs.(2.14) and (2.15) for the singleton gravity theory in the superhorizon limit of $\eta \rightarrow 0^-$. Their solutions are given by

$$\varphi_{1,0} \sim \eta^w, \quad \varphi_{2,0} \sim \eta^w \ln[-\eta] \quad (3.6)$$

with

$$w = \frac{3}{2} \left(1 - \sqrt{1 - \frac{4m^2}{9H^2}} \right). \quad (3.7)$$

The scaling of $\varphi_{a,0}$ with $a = 1, 2$ is not conventional as they transform under

$$\varphi_{1,0} \rightarrow \lambda^w \varphi_{1,0}, \quad \varphi_{2,0} \rightarrow \lambda^w [\varphi_{2,0} + \ln(\lambda) \varphi_{1,0}]. \quad (3.8)$$

A pair of dipole fields (φ_1, φ_2) is coupled to (σ_1, σ_2) -operators on the boundary (∂dS) of $\eta \rightarrow 0^-$. The explicit connection between $\varphi_{a,0}$ and σ_a is encoded by [34]

$$Z_S[\varphi_{a,0}] = Z_{\text{LCFT}}[\varphi_{a,0}], \quad (3.9)$$

$$Z_S[\varphi_{a,0}] = e^{-\delta S_S[\{\varphi_{a,0}\}]}, \quad (3.10)$$

$$Z_{\text{LCFT}}[\varphi_{a,0}] = \langle e^{-\int_{\partial\text{dS}_0} d^3x \varphi_{a,0}(\mathbf{x}) \sigma_a(\mathbf{x})} \rangle, \quad (3.11)$$

where the expectation value $\langle \dots \rangle$ is taken in the LCFT with the boundary fields $\varphi_{a,0}$ as sources. Eq.(3.9) is a statement of the dS/LCFT correspondence. Here the bulk action is given by

$$\delta S_S[\{\varphi_a\}] = - \int_{\text{dS}} d^4x \sqrt{-g} \left[\partial_\mu \varphi_1 \partial^\mu \varphi_2 + m^2 \varphi_1 \varphi_2 + \frac{\mu^2}{2} \varphi_1^2 \right]. \quad (3.12)$$

The bulk transformation (3.8) indicates that two operator σ_a of conformal dimension w transform under dilations as

$$i[D, \sigma_a] = \left(x^i \partial_i \delta_a^b + \Delta_a^b \right) \sigma_b, \quad (3.13)$$

where a dimension matrix Δ_a^b is brought to the Jordan cell form as

$$\Delta_a^b = \begin{pmatrix} w & 0 \\ 1 & w \end{pmatrix}. \quad (3.14)$$

This implies that σ_a transform under dilations of $\mathbf{x} \rightarrow \lambda \mathbf{x}$ as

$$\sigma_a(\mathbf{x}) \rightarrow \sigma'_a(\lambda \mathbf{x}) = \left(e^{\Delta \ln \lambda} \right)_a^b \sigma_b(\lambda \mathbf{x}). \quad (3.15)$$

In order to find the LCFT correlators $\langle \sigma_a(\mathbf{x})\sigma_b(\mathbf{y}) \rangle$, one might use the Ward identities for scale and special conformal transformations [9]. In this work, we wish to rederive them by using the extrapolation approach (b) (see Appendix for detail computations). The two-point functions of σ_1 and σ_2 are determined by

$${}_C\langle \sigma_1(\mathbf{x})\sigma_1(\mathbf{y}) \rangle_C = 0, \quad (3.16)$$

$${}_C\langle \sigma_1(\mathbf{x})\sigma_2(\mathbf{y}) \rangle_C = {}_C\langle \sigma_2(\mathbf{x})\sigma_1(\mathbf{y}) \rangle_C = \frac{A}{|\mathbf{x} - \mathbf{y}|^{2w}}, \quad (3.17)$$

$${}_C\langle \sigma_2(\mathbf{x})\sigma_2(\mathbf{y}) \rangle_C = \frac{A}{|\mathbf{x} - \mathbf{y}|^{2w}} \left(-2 \ln |\mathbf{x} - \mathbf{y}| + D \right). \quad (3.18)$$

Here w is a degenerate dimension of σ_1 and σ_2 . The coefficient $A = w(2w-3)$ is determined by the normalization of σ_1 and σ_2 . However, D is arbitrary. The CFT vacuum $|0\rangle_C$ is defined by three Virasoro operators $L_n|0\rangle_C = 0$ for $n = 0, \pm 1$. The highest-weight state $|\sigma_a\rangle_C = \sigma_a(0)|0\rangle_C$ for two primary fields σ_a of conformal weight $h = w/2$ is defined by

$$L_0|\sigma_1\rangle_C = h|\sigma_1\rangle_C, \quad L_0|\sigma_2\rangle_C = |\sigma_1\rangle_C + h|\sigma_2\rangle_C, \quad L_n|\sigma_a\rangle_C = 0 \text{ for } n > 0. \quad (3.19)$$

This implies that for any pair of degenerate operators σ_1 and σ_2 (logarithmic pair), the Hamiltonian (L_0) becomes non-diagonalizable which shows us a crucial difference from an ordinary CFT. Actually, Eq.(3.19) represents the CFT version of the bulk transformation (3.8). Eqs.(3.16)-(3.18) are summarized to be

$${}_C\langle \sigma_a(\mathbf{x})\sigma_b(\mathbf{y}) \rangle_C = \begin{pmatrix} 0 & \text{CFT} \\ \text{CFT} & \text{LCFT} \end{pmatrix}, \quad (3.20)$$

where CFT and LCFT represent their correlators in (3.17) and (3.18), respectively.

In order to derive the relevant correlators in momentum space, one has to use the relation

$$\frac{1}{|\mathbf{x} - \mathbf{y}|^{2w}} = \frac{\Gamma(\frac{3}{2} - w)}{4^w \pi^{3/2} \Gamma(w)} \int d^3\mathbf{k} |\mathbf{k}|^{2w-3} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{y})}, \quad (3.21)$$

where we observe an inverse-relation of exponent $2w$ between $|\mathbf{x}|$ -space and $k = |\mathbf{k}|$ -space. Finally, the correlators in momentum space are easily evaluated as [9]

$$\langle \sigma_1(\mathbf{k}_1)\sigma_1(\mathbf{k}_2) \rangle' = 0, \quad (3.22)$$

$$\langle \sigma_1(\mathbf{k}_1)\sigma_2(\mathbf{k}_2) \rangle' = \frac{A_0(w)}{k_1^{3-2w}}, \quad (3.23)$$

$$\begin{aligned} \langle \sigma_2(\mathbf{k}_1)\sigma_2(\mathbf{k}_2) \rangle' &= D \langle \sigma_1(\mathbf{k}_1)\sigma_2(\mathbf{k}_2) \rangle' + \frac{\partial}{\partial w} \langle \sigma_1(\mathbf{k}_1)\sigma_2(\mathbf{k}_2) \rangle' \\ &= \frac{A_0(w)}{k_1^{3-2w}} \left(2 \ln[k_1] + D + \frac{A_{0,w}}{A_0(w)} \right), \end{aligned} \quad (3.24)$$

where the prime ($'$) represents correlators without the $(2\pi)^3 \delta^3(\Sigma_i \mathbf{k}_i)$ and $A_{0,w} = 4w - 3$ denotes derivatives of $A_0(w) = w(2w - 3)$ with respect to w . These correlators will be compared to the power spectra in the superhorizon limit of $z \rightarrow 0$.

4 Singleton propagation in dS spacetime

In order to compute the power spectrum, we have to know the solution to singleton equations Eqs.(2.14) and (2.15) in the whole range of $\eta(z)$. For this purpose, the scalars φ_i can be expanded in Fourier modes $\phi_{\mathbf{k}}^i(\eta)$

$$\varphi_i(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \phi_{\mathbf{k}}^i(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}. \quad (4.1)$$

The first equation of (2.14) leads to

$$\left[\frac{d^2}{d\eta^2} - \frac{2}{\eta} \frac{d}{d\eta} + k^2 + \frac{m^2}{H^2} \frac{1}{\eta^2} \right] \phi_{\mathbf{k}}^1(\eta) = 0, \quad (4.2)$$

which can be further transformed into

$$\left[\frac{d^2}{d\eta^2} + k^2 - \frac{2}{\eta^2} + \frac{m^2}{H^2} \frac{1}{\eta^2} \right] \tilde{\phi}_{\mathbf{k}}^1(\eta) = 0 \quad (4.3)$$

for $\tilde{\phi}_{\mathbf{k}}^1 = a\phi_{\mathbf{k}}^1 = -\phi_{\mathbf{k}}^1/(H\eta) = \frac{k}{Hz}\phi_{\mathbf{k}}^1$. Expressing (4.3) in terms of $z = -k\eta$ leads to

$$\left[\frac{d^2}{dz^2} + 1 - \left(2 - \frac{m^2}{H^2} \right) \frac{1}{z^2} \right] \tilde{\phi}_{\mathbf{k}}^1(z) = 0. \quad (4.4)$$

Introducing $\tilde{\phi}_{\mathbf{k}}^1 = \sqrt{z}\tilde{\tilde{\phi}}_{\mathbf{k}}^1$ further, it leads to the Bessel's equation as

$$\left[\frac{d^2}{dz^2} + \frac{1}{z} \frac{d}{dz} + 1 - \frac{\nu^2}{z^2} \right] \tilde{\tilde{\phi}}_{\mathbf{k}}^1(z) = 0 \quad (4.5)$$

with the index

$$\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}. \quad (4.6)$$

The solution to (4.5) is given by the Hankel function $H_{\nu}^{(1)}$. Accordingly, one has the solution to (4.2)

$$\phi_{\mathbf{k}}^1(z) = \mathcal{C} \frac{\sqrt{z}}{a} \tilde{\phi}_{\mathbf{k}}^1 = \mathcal{C} \frac{H}{k} z^{3/2} H_{\nu}^{(1)}(z) \quad (4.7)$$

with \mathcal{C} undetermined constant. In the subhorizon limit of $z \rightarrow \infty$, Eq.(4.2) reduces to

$$\left[\frac{d^2}{dz^2} - \frac{2}{z} \frac{d}{dz} + 1 \right] \phi_{\mathbf{k},\infty}^1(z) = 0 \quad (4.8)$$

which leads the positive-frequency solution with the normalization $1/\sqrt{2k}$

$$\phi_{\mathbf{k},\infty}^1(z) = \frac{H}{\sqrt{2k^3}} (i + z) e^{iz}. \quad (4.9)$$

This is a typical mode solution of a massless scalar propagating on dS spacetime. Inspired by (4.9) and asymptotic form of $H_{\nu}^{(1)}$, $\phi_{\mathbf{k}}^1(z)$ is fixed by

$$\phi_{\mathbf{k}}^1(z) = \frac{H}{\sqrt{2k^3}} \sqrt{\frac{\pi}{2}} e^{i(\frac{\pi\nu}{2} + \frac{\pi}{4})} z^{3/2} H_{\nu}^{(1)}(z). \quad (4.10)$$

In the superhorizon limit of $z \rightarrow 0$, Eq.(4.2) takes the form

$$\left[\frac{d^2}{dz^2} - \frac{2}{z} \frac{d}{dz} + \frac{m^2}{H^2} \frac{1}{z^2} \right] \phi_{\mathbf{k},0}^1(z) = 0, \quad (4.11)$$

whose solution is

$$\phi_{\mathbf{k},0}^1(z) = \frac{H}{\sqrt{2k^3}} z^w \quad (4.12)$$

with

$$w = \frac{3}{2} - \nu. \quad (4.13)$$

On the other hand, plugging (4.1) into (2.15) leads to the degenerate fourth-order differential equation

$$\left[\eta^2 \frac{d^2}{d\eta^2} - 2\eta \frac{d}{d\eta} + k^2 \eta^2 + \frac{m^2}{H^2} \right]^2 \phi_{\mathbf{k}}^2(\eta) = 0 \quad (4.14)$$

which seems difficult to be solved directly. However, we may solve Eq.(4.14) in the two limits of subhorizon and superhorizon. In the subhorizon limit of $z \rightarrow \infty$, Eq.(4.14) takes the form

$$\left[\frac{d^4}{dz^4} + 2 \left(1 - \frac{1}{z^2} \right) \frac{d^2}{dz^2} + \frac{4}{z^3} \frac{d}{dz} + \left(1 - \frac{2}{z^2} \right) \right] \phi_{\mathbf{k},\infty}^2 = 0. \quad (4.15)$$

whose direct solution is given by

$$\phi_{\mathbf{k},\infty}^{2,d} = \left[\tilde{c}_2(i+z) + \tilde{c}_1 \left(2i + (z-i)e^{-2iz} \text{Ei}(2iz) \right) \right] e^{iz} \quad (4.16)$$

with two coefficients \tilde{c}_1 and \tilde{c}_2 . The c.c. of $\phi_{\mathbf{k},\infty}^{2,d}$ is a solution to (4.15) too. Here $\text{Ei}(2iz)$ is the exponential integral function defined by [35]

$$\text{Ei}(2iz) = \text{Ci}(2z) + i\text{Si}(2z) + i\frac{\pi}{2}, \quad (4.17)$$

where the cosine-integral and sine-integral functions are given by

$$\text{Ci}(2z) = \int_0^{2z} \frac{\cos t}{t} dt, \quad \text{Si}(2z) = \int_0^{2z} \frac{\sin t}{t} dt. \quad (4.18)$$

We note that $\text{Ei}(2iz)$ satisfies the fourth-order equation

$$\begin{aligned} (z-i)z^3 \frac{d^4 \text{Ei}}{dz^4} - 4iz^4 \frac{d^3 \text{Ei}}{dz^3} + 2z(i-z-4iz^2-2z^3) \frac{d^2 \text{Ei}}{dz^2} \\ - 4(i-z-iz^2+2z^3) \frac{d \text{Ei}}{dz} - 8e^{2iz} = 0. \end{aligned} \quad (4.19)$$

However, we wish to point out that the direct solution (4.16) is not suitable for choosing the Bunch-Davies vacuum to give quantum fluctuations. In order to find an appropriate solution, we note that $(\bar{\nabla}^2 - m^2)\varphi_2 = \mu^2\varphi_1$ in (2.14) reduces to in the subhorizon limit

$$\left[\frac{d^2}{dz^2} - \frac{2}{z} \frac{d}{dz} + 1 \right] \phi_{\mathbf{k},\infty}^2(z) = 0, \quad (4.20)$$

whose solution is

$$\phi_{\mathbf{k},\infty}^2(z) = \tilde{c}_2(i+z)e^{iz}. \quad (4.21)$$

We note that $\phi_{\mathbf{k},\infty}^2(z)$ is included as the first term of (4.16) [as a solution to the fourth-order equation (4.15)].

On the other hand, Eq.(4.14) takes the form in the superhorizon limit of $z \rightarrow 0$ as

$$\left[z^2 \frac{d^2}{dz^2} - 2z \frac{d}{dz} + \frac{m^2}{H^2} \right]^2 \phi_{\mathbf{k},0}^2(z) = 0 \quad (4.22)$$

whose solution is given by

$$\phi_{\mathbf{k},0}^2(z) \propto z^w \ln z. \quad (4.23)$$

This also satisfies

$$(-H^2) \left[z^2 \frac{d^2}{dz^2} - 2z \frac{d}{dz} + \frac{m^2}{H^2} \right] \phi_{\mathbf{k},0}^2(z) = \mu^2 \phi_{\mathbf{k},0}^1(z) \quad (4.24)$$

for $\mu^2 = (3 - 2w)H^2$ which is the superhorizon limit of Eq.(2.14). The presence of “ $\ln z$ ” implies that (4.23) is a solution to the fourth-order equation (4.22)

Finally, the trick used in [6] implies that one may solve (4.14) directly by differentiating $(\bar{\nabla}^2 - m^2)\varphi_1 = 0$ with respect to m^2 . The explicit steps are given by

$$\frac{d}{dm^2} \times \left(-z^2 H^2 \frac{d^2}{dz^2} + 2z H^2 \frac{d}{dz} - z^2 H^2 - m^2 \right) \phi_{\mathbf{k}}^1(z) = 0 \quad (4.25)$$

$$\rightarrow \left(-z^2 H^2 \frac{d^2}{dz^2} + 2z H^2 \frac{d}{dz} - z^2 H^2 - m^2 \right) \frac{d}{dm^2} \phi_{\mathbf{k}}^1(z) = \phi_{\mathbf{k}}^1(z) \quad (4.26)$$

$$\leftrightarrow \left(-z^2 H^2 \frac{d^2}{dz^2} + 2z H^2 \frac{d}{dz} - z^2 H^2 - m^2 \right) \phi_{\mathbf{k}}^2(z) = \mu^2 \phi_{\mathbf{k}}^1(z) \quad (4.27)$$

which provides a way to obtain $\phi_{\mathbf{k}}^2(z)$ from $\phi_{\mathbf{k}}^1(z)$ as

$$\phi_{\mathbf{k}}^2(z) = \mu^2 \frac{d}{dm^2} \phi_{\mathbf{k}}^1(z). \quad (4.28)$$

We note that (4.14) can be obtained by acting $(\bar{\nabla}^2 - m^2)$ on (4.27). Explicitly, $\frac{d}{dm^2} \phi_{\mathbf{k}}^1(z)$ is computed to be

$$\begin{aligned} \frac{d}{dm^2} \phi_{\mathbf{k}}^1(z) = & -\frac{1}{2\nu H \sqrt{2k^3}} \sqrt{\frac{\pi}{2}} e^{i(\frac{\pi\nu}{2} + \frac{\pi}{4})} z^{3/2} \left\{ \pi \left(\frac{i}{2} - \cot[\nu\pi] \right) H_{\nu}^{(1)} + i \csc[\nu\pi] \times \right. \\ & \left. \left(e^{-\nu\pi i} \frac{\partial}{\partial \nu} J_{\nu} - \frac{\partial}{\partial \nu} J_{-\nu} - \pi i e^{-\nu\pi i} J_{\nu} \right) \right\}, \end{aligned} \quad (4.29)$$

where

$$\frac{\partial}{\partial \nu} J_{\nu}(z) = J_{\nu} \ln \left[\frac{z}{2} \right] - \left(\frac{z}{2} \right)^{\nu} \sum_{k=0}^{\infty} (-1)^k \frac{\psi(\nu + k + 1)}{\Gamma(\nu + k + 1)} \frac{\left(\frac{z^2}{4} \right)^k}{k!} \quad (4.30)$$

with the digamma function $\psi(x) = \partial \ln[\Gamma(x)]/\partial x$. Here we observe the appearance of $\ln[z]$ -term. It turns out that $\phi_{\mathbf{k}}^2(z)$ takes the form when considering $J_{\pm\nu} \rightarrow \Gamma(\pm\nu + 1)^{-1}(z/2)^{\pm\nu}$ in the superhorizon limit of $z \rightarrow 0$ as

$$\phi_{\mathbf{k}}^2(z) \sim z^w \ln[z], \quad (4.31)$$

which recovers (4.23). We mention that $\frac{\partial}{\partial\nu}J_{-\nu}$ in (4.29) is dominant because it behaves as $z^{-\nu} \ln[z]$ in the superhorizon limit of $z \rightarrow 0$. However, we do not recover its asymptotic form (4.21) in the subhorizon limit of $z \rightarrow \infty$. Hence, it is not easy to obtain a full solution $\phi_{\mathbf{k}}^2(z)$ to (4.14) by the trick used in [6]. Fortunately, its superhorizon-limit solution (4.23) could be found by this trick.

5 Power spectra

The power spectrum is defined by the two-point function which could be computed when one chooses the Bunch-Davies (BD) vacuum state $|0\rangle_{\text{BD}}$ in the subhorizon limit (∂dS_∞) of $\eta \rightarrow -\infty (z \rightarrow \infty)$ [14]. The defining relation is given by

$${}_{\text{BD}}\langle 0 | \mathcal{F}(\eta, \mathbf{x}) \mathcal{F}(\eta, \mathbf{y}) | 0 \rangle_{\text{BD}} = \int d^3k \frac{\mathcal{P}_{\mathcal{F}}}{4\pi k^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}, \quad (5.1)$$

where \mathcal{F} represents singleton and tensor and $k = \sqrt{\mathbf{k}\cdot\mathbf{k}}$ is the comoving wave number. Quantum fluctuations were created on all length scales with wave number k . Cosmologically relevant fluctuations start their lives inside the Hubble radius which defines the subhorizon: $k \gg aH$. On later, the comoving Hubble radius $1/(aH)$ shrinks during inflation while keeping the wavenumber k constant. Eventually, all fluctuations exit the comoving Hubble radius, they reside on the superhorizon region of $k \ll aH$ after horizon crossing.

In general, one may compute the power spectrum of scalar and tensor by taking the BD vacuum. In the dS inflation, we choose the subhorizon limit of $z \rightarrow \infty$ to define the BD vacuum. This implies that in the infinite past of $\eta \rightarrow -\infty (z \rightarrow \infty)$, all observable modes had time-independent frequencies $\omega = k$ and the Mukhanov-Sasaki equation reduces to $\mathcal{F}_{\mathbf{k},\infty}'' + k^2 \mathcal{F}_{\mathbf{k},\infty} \approx 0$ whose positive solution is given by $\mathcal{F}_{\mathbf{k},\infty} = e^{-ik\eta}/\sqrt{2k} = e^{iz}/\sqrt{2k}$. This defines a preferable set of mode functions and a unique physical vacuum, the BD vacuum $|0\rangle_{\text{BD}}$.

On the other hand, we choose the superhorizon region of $z \ll 1$ to get a finite form of the power spectrum which stays alive after decaying. For example, fluctuations of a massless scalar ($\bar{\nabla}^2 \delta\phi = 0$) and tensor ($\bar{\nabla}^2 h_{ij} = 0$) with different normalization originate on subhorizon scales and they propagate for a long time on superhorizon scales. This can be checked by computing their power spectra given by

$$\mathcal{P}_{\delta\phi} = \frac{H^2}{(2\pi)^2} [1 + z^2], \quad (5.2)$$

$$\mathcal{P}_{\text{h}} = 2 \times \left(\frac{2}{M_{\text{P}}}\right)^2 \times \mathcal{P}_{\delta\phi} = \frac{2H^2}{\pi^2 M_{\text{P}}^2} [1 + z^2]. \quad (5.3)$$

In the limit of $z \rightarrow 0$, they are finite as

$$\mathcal{P}_{\delta\phi,0} = \frac{H^2}{(2\pi)^2}, \quad \mathcal{P}_{h,0} = \frac{2H^2}{\pi^2 M_{\text{P}}^2}. \quad (5.4)$$

Accordingly, it would be very interesting to check what happens when one computes the power spectra for the dipole pair (singleton) generated from during the dS inflation in the framework of the singleton gravity theory.

To compute the power spectrum, we have to know the commutation relations and the Wronskian conditions. The canonical conjugate momenta are given by

$$\pi_1 = a^2 \frac{d\varphi_2}{d\eta}, \quad \pi_2 = a^2 \frac{d\varphi_1}{d\eta}. \quad (5.5)$$

The canonical quantization is accomplished by imposing equal-time commutation relations:

$$[\hat{\varphi}_1(\eta, \mathbf{x}), \hat{\pi}_1(\eta, \mathbf{y})] = i\delta^3(\mathbf{x} - \mathbf{y}), \quad [\hat{\varphi}_2(\eta, \mathbf{x}), \hat{\pi}_2(\eta, \mathbf{y})] = i\delta^3(\mathbf{x} - \mathbf{y}). \quad (5.6)$$

The two operators $\hat{\varphi}_1$ and $\hat{\varphi}_2$ are expanded in terms of Fourier modes as [27, 28, 36]

$$\hat{\varphi}_1(z, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k N \left[\left(i\hat{c}_1(\mathbf{k})\phi_{\mathbf{k}}^1(z)e^{i\mathbf{k}\cdot\mathbf{x}} \right) + \text{h.c.} \right], \quad (5.7)$$

$$\hat{\varphi}_2(z, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \tilde{N} \left[\left(\hat{c}_2(\mathbf{k})\phi_{\mathbf{k}}^1(z) + \hat{c}_1(\mathbf{k})\phi_{\mathbf{k}}^2(z) \right) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right] \quad (5.8)$$

with N and \tilde{N} the normalization constants. Plugging (5.7) and (5.8) into (5.6) determines the relation of normalization constants as $N\tilde{N} = 1/2k$ and commutation relations between $\hat{c}_a(\mathbf{k})$ and $\hat{c}_b^\dagger(\mathbf{k}')$ as

$$[\hat{c}_a(\mathbf{k}), \hat{c}_b^\dagger(\mathbf{k}')] = 2k \begin{pmatrix} 0 & -i \\ i & 1 \end{pmatrix} \delta^3(\mathbf{k} - \mathbf{k}') \quad (5.9)$$

which reflects the quantization of singleton. Here, the commutation relation of $[\hat{c}_2(\mathbf{k}), \hat{c}_2^\dagger(\mathbf{k}')]$ is implemented by the following Wronskian condition with (4.9) and $\tilde{c}_2 = -iH/(2\sqrt{2k^3})$ in (4.21):

$$a^2 \left(\phi_{\mathbf{k},\infty}^1 \frac{d\phi_{\mathbf{k},\infty}^{2*}}{dz} - \phi_{\mathbf{k},\infty}^{2*} \frac{d\phi_{\mathbf{k},\infty}^1}{dz} + \phi_{\mathbf{k},\infty}^{1*} \frac{d\phi_{\mathbf{k},\infty}^2}{dz} - \phi_{\mathbf{k},\infty}^2 \frac{d\phi_{\mathbf{k},\infty}^{1*}}{dz} \right) = \frac{1}{k}. \quad (5.10)$$

It is important to note that the commutation relations (5.9) were used to derive the power spectra of conformal gravity [37]. On the other hand, if one uses the solution $\phi_{\mathbf{k},\infty}^1$ (4.9) and $\phi_{\mathbf{k},\infty}^{2,d}$ (4.16), the Wronskian condition leads to

$$\begin{aligned} & a^2 \left(\phi_{\mathbf{k},\infty}^1 \frac{d\phi_{\mathbf{k},\infty}^{2,d*}}{dz} - \phi_{\mathbf{k},\infty}^{2,d*} \frac{d\phi_{\mathbf{k},\infty}^1}{dz} + \phi_{\mathbf{k},\infty}^{1*} \frac{d\phi_{\mathbf{k},\infty}^{2,d}}{dz} - \phi_{\mathbf{k},\infty}^{2,d} \frac{d\phi_{\mathbf{k},\infty}^{1*}}{dz} \right) \\ &= -\sqrt{k} \frac{1}{2H} \left[2i(-\tilde{c}_2 + \tilde{c}_2^*) + (\tilde{c}_1 + \tilde{c}_1^*) \left(\frac{1}{z^3} + \frac{3}{z} \right) \right] \end{aligned} \quad (5.11)$$

which cannot be independent of z unless $\tilde{c}_1 = \tilde{c}_1^* = 0$. This explains why the direct solution $\phi_{\mathbf{k},\infty}^{2,d}$ (4.16) is not suitable for choosing the Bunch-Davies vacuum in the subhorizon limit. At this stage, we wish to mention when do the fluctuations of singleton become classical. The commutators in (5.6) commute on the superhorizon region of $z < 1$ after horizon crossing.

We are ready to compute the power spectrum of the dipole pair defined by

$${}_{\text{BD}}\langle 0|\hat{\varphi}_a(\eta, \mathbf{x})\hat{\varphi}_b(\eta, \mathbf{y})|0\rangle_{\text{BD}} = \int d^3k \frac{\mathcal{P}_{ab}}{4\pi k^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}. \quad (5.12)$$

Here we choose the BD vacuum $|0\rangle_{\text{BD}}$ by imposing $\hat{c}_a(\mathbf{k})|0\rangle_{\text{BD}} = 0$. On the other hand, the cosmological correlator defined in momentum space are related to the power spectra as [14]

$$\langle \phi_{\mathbf{k}}^a \phi_{\mathbf{k}'}^b \rangle = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} P_{ab}(k). \quad (5.13)$$

Since the singleton theory is quite different from the two-free scalar theory, we explain what the BD vacuum is. For this purpose, we remind the reader that the Gupta-Bleuler condition of $B^+(\mathbf{x})|\text{phys}\rangle = 0$ where B is a conjugate momentum of scalar photon A_0 was introduced to extract the physical states of transverse photons A_1 and A_2 by confining scalar photon A_0 and longitudinal photon A_3 as members of quartet [38, 39]. For this purpose, we note that the dipole pair (φ_1, φ_2) is turned into the zero-norm state by making use of the BRST transformation in Minkowski spacetime [40]. We suggest that if the dS/LCFT correspondence works, the boundary logarithmic operator σ_2 is related to the negative-norm state of φ_2 . In order to remove the negative-norm state, we impose the subsidiary condition as $\varphi_1^+(\mathbf{x})|\text{phys}\rangle = 0$ where $\varphi_1^+(\mathbf{x})$ is the positive-frequency part of the field operator. Then, the physical space ($|\text{phys}\rangle$) will not include any φ_2 -particle state. This corresponds to the dipole mechanism to cancel the negative-norm state. Here, the subsidiary condition of $\varphi_1^+(\mathbf{x})|\text{phys}\rangle = 0$ is translated into $\hat{c}_1(\mathbf{k})|\text{phys}\rangle = 0$ which shares a property of the BD vacuum $|0\rangle_{\text{BD}}$ defined by $\hat{c}_1(\mathbf{k})|0\rangle_{\text{BD}} = 0$, in addition to $\hat{c}_2(\mathbf{k})|0\rangle_{\text{BD}} = 0$.

The tensor power spectrum for φ_1 is given as

$$\mathcal{P}_{11} = 0 \quad (5.14)$$

when one used the unconventional commutation relation $[\hat{c}_1(\mathbf{k}), \hat{c}_1^\dagger(\mathbf{k}')] = 0$.

On the other hand, it turns out that the power spectrum of φ_2 is defined by

$$\begin{aligned} \mathcal{P}_{22} &\equiv \mathcal{P}_{22}^{(1)} + \mathcal{P}_{22}^{(2)} \\ &= \frac{k^3}{2\pi^2} \left(\left| \phi_{\mathbf{k}}^1 \right|^2 + i(\phi_{\mathbf{k}}^1 \phi_{\mathbf{k}}^{2*} - \phi_{\mathbf{k}}^2 \phi_{\mathbf{k}}^{1*}) \right), \end{aligned} \quad (5.15)$$

where $\mathcal{P}_{22}^{(1,2)}$ denote the (first, second) term in (5.15) and we fixed $\tilde{N} = 1/\sqrt{2k}$. Note that $\mathcal{P}_{22}^{(1)}$ can be written as

$$\mathcal{P}_{22}^{(1)} = \frac{k^3}{2\pi^2} \left| \phi_{\mathbf{k}}^1 \right|^2 = \frac{H^2}{8\pi} z^3 \left| e^{i(\frac{\pi\nu}{2} + \frac{\pi}{4})} H_\nu^{(1)}(z) \right|^2. \quad (5.16)$$

In the superhorizon limit of $z \rightarrow 0$, the power spectrum takes the form

$$\mathcal{P}_{22}^{(1)} \Big|_{z \rightarrow 0} = \left(\frac{H}{2\pi} \right)^2 \left(\frac{\Gamma(\nu)}{\Gamma(3/2)} \right)^2 \left(\frac{z}{2} \right)^{2w} \equiv \xi^2 z^{2w}, \quad \xi^2 = \frac{1}{2^{2w}} \left(\frac{H}{2\pi} \right)^2 \left(\frac{\Gamma(\nu)}{\Gamma(3/2)} \right)^2. \quad (5.17)$$

which implies that $\mathcal{P}_{22}^{(1)}$ approaches zero as $z \rightarrow 0$. In the massless case of $m^2 = 0$ ($\nu = 3/2, w = 0$), $\mathcal{P}_{22}^{(1)}$ leads to the power spectrum $\mathcal{P}_{\delta\phi} = (H/2\pi)^2$ in (5.2) for a massless scalar.

It is important to note that in the superhorizon limit of $z \rightarrow 0$, $\mathcal{P}_{22}^{(2)}$ is given by

$$\mathcal{P}_{22}^{(2)} \sim 2\xi^2 z^{2w} \ln[z], \quad (5.18)$$

which implies that $\mathcal{P}_{22}^{(2)}$ approaches zero as $z \rightarrow 0$. In deriving (5.18), ξ denotes a real quantity given by $\phi_{\mathbf{k}}^1 = -i\xi z^w$ and $\phi_{\mathbf{k}}^2 \sim \xi z^w \ln[z]$. We mention that the remaining power spectra \mathcal{P}_{12} and \mathcal{P}_{21} take the same form as $\mathcal{P}_{22}^{(1)}$

$$\begin{aligned} \mathcal{P}_{12} &= \mathcal{P}_{21} = \frac{k^3}{2\pi^2} \left| \phi_{\mathbf{k}}^1 \right|^2 \\ &= \mathcal{P}_{22}^{(1)}, \end{aligned} \quad (5.19)$$

where we fixed $N = 1/\sqrt{2k}$.

Finally, we obtain the power spectra of singleton in the superhorizon limit of $z \rightarrow 0$

$$\mathcal{P}_{ab,0}(z) \sim \xi^2 \begin{pmatrix} 0 & z^{2w} \\ z^{2w} & z^{2w}(1 + 2\ln[z]) \end{pmatrix}. \quad (5.20)$$

Its explicit form is given by

$$\mathcal{P}_{ab,0}(k, \eta) \sim \xi^2 \begin{pmatrix} 0 & (-k\eta)^{2w} \\ (-k\eta)^{2w} & (-k\eta)^{2w}(1 + 2\ln[-k\eta]) \end{pmatrix}. \quad (5.21)$$

For $\eta = -\epsilon(0 < \epsilon \ll 1)$ near $\eta = 0^-$ [41], (5.21) takes the form

$$\mathcal{P}_{ab,0}(k, -\epsilon) \sim \xi^2 \begin{pmatrix} 0 & (\epsilon k)^{2w} \\ (\epsilon k)^{2w} & (\epsilon k)^{2w}(1 + 2\ln[\epsilon k]) \end{pmatrix}. \quad (5.22)$$

Interestingly, $k^{-3}\mathcal{P}_{ab,0}(k, -1)$ has the same form as the momentum correlators of LCFT $\langle \sigma_a(k)\sigma_b(-k) \rangle$ with $D = (2w-1)(w-3)/(w(2w-3))$ in (3.22)-(3.24). This may show how the dS/LCFT correspondence works for deriving the power spectra in the superhorizon limit. For a light singleton with $m^2 \ll H^2$, one has $w \simeq \frac{m^2}{3H^2}$. Hence, these power spectra are given by

$$\mathcal{P}_{ab,0} \Big|_{\frac{m^2}{H^2} \ll 1}(k, -\epsilon) \propto \begin{pmatrix} 0 & (\epsilon k)^{\frac{2m^2}{3H^2}} \\ (\epsilon k)^{\frac{2m^2}{3H^2}} & (\epsilon k)^{\frac{2m^2}{3H^2}}(1 + 2\ln[\epsilon k]) \end{pmatrix} \quad (5.23)$$

whose spectral indices are given by

$$n_{ab,0} \Big|_{\frac{m^2}{H^2} \ll 1}(k, -\epsilon) - 1 = \frac{d \ln \mathcal{P}_{ab,0} \Big|_{\frac{m^2}{H^2} \ll 1}(k, -\epsilon)}{d \ln k} = \begin{pmatrix} 0 & \frac{2m^2}{3H^2} \\ \frac{2m^2}{3H^2} & \frac{2m^2}{3H^2} + \frac{2}{(1+2\ln[\epsilon k])} \end{pmatrix}. \quad (5.24)$$

We observe here that $n_{ab,0}|_{\frac{m^2}{H^2} \ll 1}$ gets a new contribution $\frac{2}{(1+2\ln[\epsilon k])}$ from the due to the logarithmic short distance singularity. Also, we observe that $\mathcal{P}_{22,0}|_{\frac{m^2}{H^2} \ll 1}(k, -\epsilon) < 0$ for $\epsilon k < 0.607$. There is no such condition for a massive scalar propagating on the dS spacetime.

At this stage, we briefly mention how to resolve the ϵ -dependence. To compute the power spectra and spectral indices correctly, one has to choose a proper slice near $\eta = 0^-$. This may be done by taking $\eta = -\epsilon$ firstly, and letting $\epsilon \rightarrow 0$ on later. We note that the ϵ -dependence appears in the power spectra (5.22) and spectral indices (5.24). As was shown in the dS/CFT correspondence [41], the cut-off ϵ acts like a renormalization scale which is well-known from the UV CFT renormalization theory. The cosmic evolution can be seen as a reversed renormalization group flow, from the IR fixed point (big bang) of the dual CFT to the UV fixed point (late times) of the dual CFT theory [42]. Inflation occurs at a certain intermediate stage during the renormalization group flow. This is called as dS holography. Accordingly, in order to obtain the ϵ -independent power spectra and spectral indices, we should introduce proper counter terms to renormalize the power spectra and spectral indices.

In the massless singleton of $m^2 = 0$ ($\nu = 3/2, w = 0$), the corresponding power spectra take the form

$$\mathcal{P}_{ab,0}|_{m^2 \rightarrow 0} = \left(\frac{H}{2\pi}\right)^2 \begin{pmatrix} 0 & 1 \\ 1 & 1 + 2\ln[z] \end{pmatrix} \quad (5.25)$$

in the superhorizon limit. Here, we note that $\mathcal{P}_{12,0}|_{m^2 \rightarrow 0}$ is just the power spectrum of a massless scalar $\mathcal{P}_{\delta\phi,0}$ (5.4) in the superhorizon limit.

6 Discussions

In this work, we have obtained the power spectra of singleton generated during the dS inflation. Even though we did not know a complete solution of $\phi_{\mathbf{k}}^2$ to the degenerate fourth-order equation (4.14) in whole region, we have obtained the power spectra which show that the dS/LCFT correspondence plays an important role in determining the power spectra in the superhorizon limit. Considering (5.13) and (5.21), one has $k^{-3}\mathcal{P}_{ab,0}(k, -1) \propto \langle \phi_{\mathbf{k}}^a \phi_{-\mathbf{k}}^b \rangle$. Hence, the cosmological correlators $\langle \phi_{\mathbf{k}}^a \phi_{-\mathbf{k}}^b \rangle$ are directly proportional to the momentum LCFT-correlators $\langle \sigma_a(k) \sigma_b(-k) \rangle$ in (3.22)-(3.24). Here we note that LCFT correlators were derived from the “extrapolate” dictionary (b). This is compared to the “differentiate” dictionary where (3.5) states that the cosmological correlator was inversely proportional to the CFT correlator [26]. Furthermore, we have computed the spectral indices (5.24) for a light singleton which contains a logarithmic correction, in compared to the massive scalar.

In computing the power spectra, we have used two vacua located at $z = \infty$ (∂dS_∞) and $z = 0$ (∂dS_0): the BD vacuum $|0\rangle_{\text{BD}}$ in the subhorizon limit of $z \rightarrow \infty$ ($\eta \rightarrow -\infty$) and the CFT vacuum $|0\rangle_{\text{C}}$ to define the correlators of operators σ_a in the superhorizon limit of $z \rightarrow 0$ ($\eta \rightarrow 0^-$). The BD vacuum $|0\rangle_{\text{BD}}$ is annihilated by the two lowering operators as $c_a(\mathbf{k})|0\rangle_{\text{BD}} = 0$, and it relates to the $|\text{phys}\rangle$ which annihilates the negative norm state in the quantum electrodynamics. This is because the singleton theory is not a two-free scalar theory. In addition, the commutation relations (5.9) designed for the singleton quantization

played an important role to derive the power spectra in the superhorizon limit. On the other hand, the CFT vacuum $|0\rangle_C$ was defined by imposing the Virasoro operators $L_n|0\rangle_C = 0$ for $n = 0, \pm 1$. The highest-weight state $|\Phi\rangle_C = \Phi(0)|0\rangle_C$ for any primary field Φ of conformal weight h is defined by $L_0|\Phi\rangle_C = h|\Phi\rangle_C$ and $L_n|\Phi\rangle_C = 0$ for $n > 0$.

Consequently, we have derived the power spectra and spectral indices of singleton in the superhorizon limit by using two boundary conditions at the infinite past ($\eta = -\infty$) and infinite future ($\eta = 0^-$) where the BD vacuum was taken on the former time, while the CFT vacuum was employed on the latter time. The dS/LCFT correspondence was firstly realized as the computation of singleton power spectra. Since the LCFT as dual to the singleton suffers from the non-unitarity (for example, $\mathcal{P}_{22,0}|_{\frac{m^2}{H^2} \ll 1}(k, -\epsilon) < 0$ for $\epsilon k < 0.607$), a truncation mechanism will be introduced to cure the non-unitarity in dS spacetime [8, 40, 43]. However, there remains nothing ($\sigma_{11} = 0$) for the rank-2 LCFT dual to singleton after truncating (3.20). If one considers three-coupled scalar theory instead of singleton, its dual correlators will be not a 2×2 matrix (3.20) but a 3×3 matrix of

$$\tilde{\sigma}_{ab} \propto \begin{pmatrix} 0 & 0 & \text{CFT} \\ 0 & \text{CFT} & \text{LCFT} \\ \text{CFT} & \text{LCFT} & \text{LCFT}^2 \end{pmatrix}. \quad (6.1)$$

The truncation process be carried out by throwing all terms which generate the third column and row of (6.1). Actually, this corresponds to finding a unitary CFT. We point out that a unitary CFT ($\tilde{\sigma}_{22}$) obtained after truncation is nothing but an ordinary CFT.

Finally, let us ask how could this scenario account for cosmological observables like the amplitude of the power spectrum and the tensor-to-scalar ratio in the cosmic microwave background. In this work, we have chosen the dS inflation with $\dot{\phi}_1 = \dot{\phi}_2 = 0$ instead of the slow-roll (dS-like) inflation for simplicity. If we choose the slow-roll inflation, then the Einstein equation takes the form of $G_{\mu\nu} = T_{\mu\nu}/M_{\text{P}}^2$ which provides the energy density $\rho = \dot{\phi}_1\dot{\phi}_2 + (m^2\phi_1\phi_2 + \mu^2\phi_1^2/2)$ and the pressure $p = \dot{\phi}_1\dot{\phi}_2 - (m^2\phi_1\phi_2 + \mu^2\phi_1^2/2)$. The first and second Friedmann equations are given by $H^2 = \frac{\rho}{3M_{\text{P}}^2}$ and $\dot{H} = -\frac{\rho+p}{2M_{\text{P}}^2}$. Also, their scalar equations are given by $\ddot{\phi}_1 + 3H\dot{\phi}_1 + m^2\phi_1 = 0$ and $\ddot{\phi}_2 + 3H\dot{\phi}_2 + m^2\phi_2 = -\mu^2\phi_1$ which are combined to give $(\frac{d^2}{dt^2} + 3H\frac{d}{dt} + m^2)^2\phi_2 = 0$. However, it requires a formidable task to perform its cosmological perturbations around the slow-roll inflation instead of the dS inflation. Hence, we wish to remain ‘‘cosmological perturbations of singleton’’ as a future work by answering to the question how could this theory account for the observed cosmological parameters in the cosmic microwave background.

On the other hand, one may consider the holographic inflation and thus, the dS/CFT correspondence determines the tensor central charge. If one accepts holographic inflation such that the dS inflation era of our universe is approximately described by a dual CFT_3 living on the spatial slice at the end of inflation, the BICEP2 results might determine the central charge $c_{\text{T}} = 1.2 \times 10^9$ of the CFT_3 [44]. This is because every CFT_3 has a transverse-traceless tensor T_{ij} with two DOF which satisfies $\langle T_{ij}(\mathbf{x})T_{kl}(\mathbf{0}) \rangle = \frac{c_{\text{T}}}{|\mathbf{x}|^6}I_{ij,kl}(\mathbf{x})$. Since a single complex scalar ψ represents two polarization modes of the graviton, its tensor correlator in momentum space is defined by $\langle \psi_{\mathbf{k}}\psi_{\mathbf{k}'} \rangle = (2\pi)^3\delta^3(\mathbf{k} + \mathbf{k}')\frac{2\pi^2}{k^3}\frac{c_{\text{T}}}{2}$ which

determines the tensor power spectrum $\mathcal{P}_T = 2\left(\frac{Ht_P}{\pi}\right)^2 = \mathcal{P}_{h,0}$ in (5.4). This was determined to be 5×10^{-10} by BICEP2 [12]. Also, its improvement of energy-momentum tensor was reported in [45] by including a curvature coupling of $\zeta\phi^2 R$. As a result, if one uses the critical gravity including curvature squared terms to describe the holographic inflation, the dS/LCFT picture for tensor modes would play a role in determining other cosmological observables.

Appendix: LCFT correlators from “extrapolate” dictionary

In this appendix, we derive the LCFT correlators by making use of the extrapolation approach (b) in the superhorizon limit. For this purpose, we consider the Green’s function for a massive scalar propagating on dS spacetime

$$G_0(\eta, \mathbf{x}; \eta', \mathbf{y}) = \frac{H^2}{16\pi} \Gamma(\Delta_+) \Gamma(\Delta_-) {}_2F_1(\Delta_+, \Delta_-, 2; 1 - \frac{\xi}{4}) \quad (6.2)$$

with $\xi = \frac{-(\eta - \eta')^2 + |\mathbf{x} - \mathbf{y}|^2}{\eta\eta'}$. Taking a transformation form of hypergeometric function

$${}_2F_1(\Delta_+, \Delta_-, 2; 1 - \frac{\xi}{4}) = \left(\frac{4}{\xi}\right)^{\Delta_-} {}_2F_1\left(\Delta_-, 2 - \Delta_+, 2; \frac{1 - \frac{\xi}{4}}{-\frac{\xi}{4}}\right), \quad (6.3)$$

we obtain the asymptotic form for $\Delta_- = w$

$$\lim_{\eta, \eta' \rightarrow 0} (\eta\eta')^{-w} G_0(\eta, \mathbf{x}; \eta', \mathbf{y}) \propto \frac{1}{|\mathbf{x} - \mathbf{y}|^{2w}}, \quad (6.4)$$

which corresponds to LCFT correlators ${}_e\langle \mathcal{O}_1(\mathbf{x}) \mathcal{O}_2(\mathbf{y}) \rangle_e = {}_e\langle \mathcal{O}_2(\mathbf{x}) \mathcal{O}_1(\mathbf{y}) \rangle_e$. Furthermore, the Green’s function G_1 is derived by taking derivative with respect to w as

$$G_1 = \frac{d}{dw} G_0 = \left(\frac{4}{\xi}\right)^w \left(-\ln\left[\frac{\xi}{4}\right] + \frac{1}{F} \frac{\partial F}{\partial w} \right) F, \quad (6.5)$$

where F denotes $F = H^2 \Gamma(3 - w) \Gamma(w) {}_2F_1(w, w - 1, 2; 1 - 4/\xi)/(16\pi)$. It turns out that its asymptotic form is given by

$$\lim_{\eta, \eta' \rightarrow 0} (\eta\eta')^{-w} G_1(\eta, \mathbf{x}; \eta', \mathbf{y}) \propto \frac{1}{|\mathbf{x} - \mathbf{y}|^{2w}} \left(-2 \ln |\mathbf{x} - \mathbf{y}| + \zeta_1 \right), \quad (6.6)$$

where ζ_1 is some constant and (6.6) corresponds to ${}_e\langle \mathcal{O}_2(\mathbf{x}) \mathcal{O}_2(\mathbf{y}) \rangle_e$.

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